ccp and event structures: towards a true concurrency semantics of stochastic, real-time ccp

Camilo Rueda [1]

[1] Universidad Javeriana-Cali

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The ccp model

- A collection of agents
- Agents compute partial information
- Information is recorded in a store
- the store grows monotonically
- Agents operate concurrently
- they synchronize by shared variables

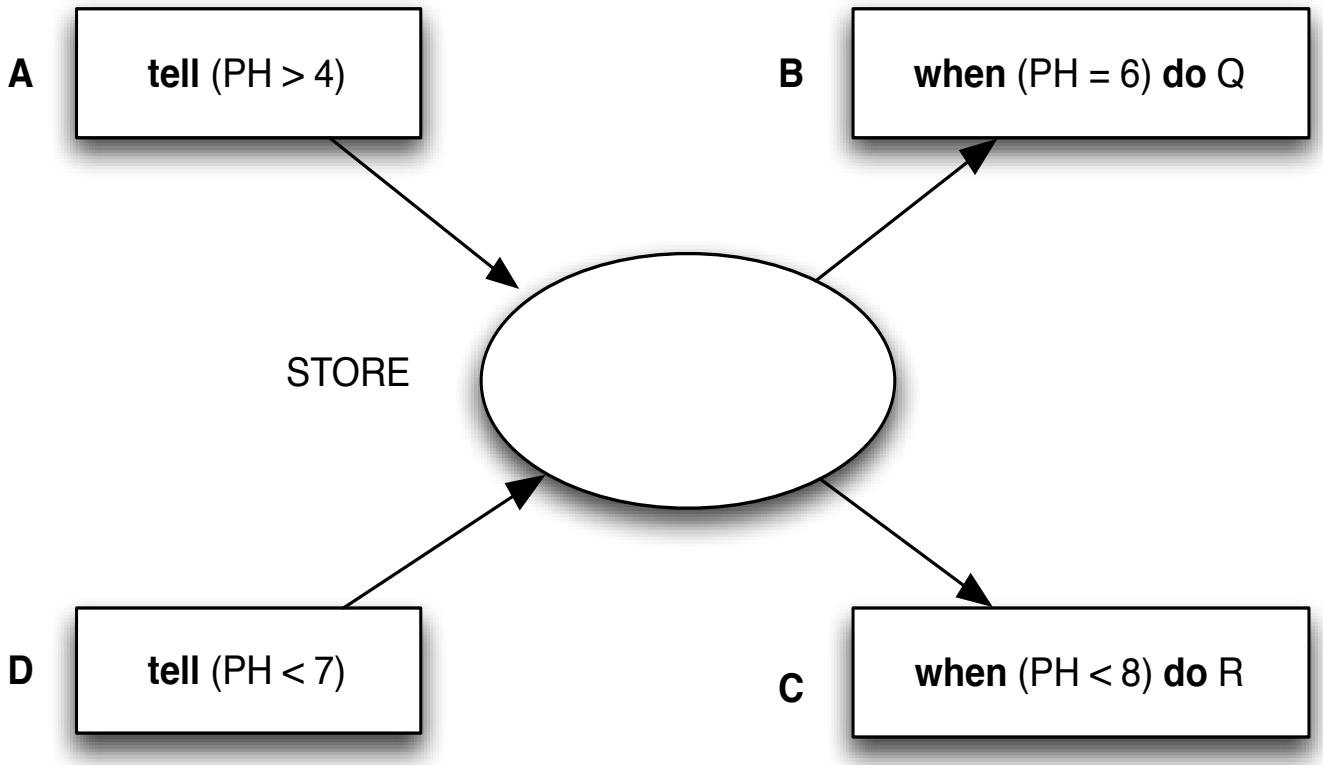
Agents

Two kinds of agents:

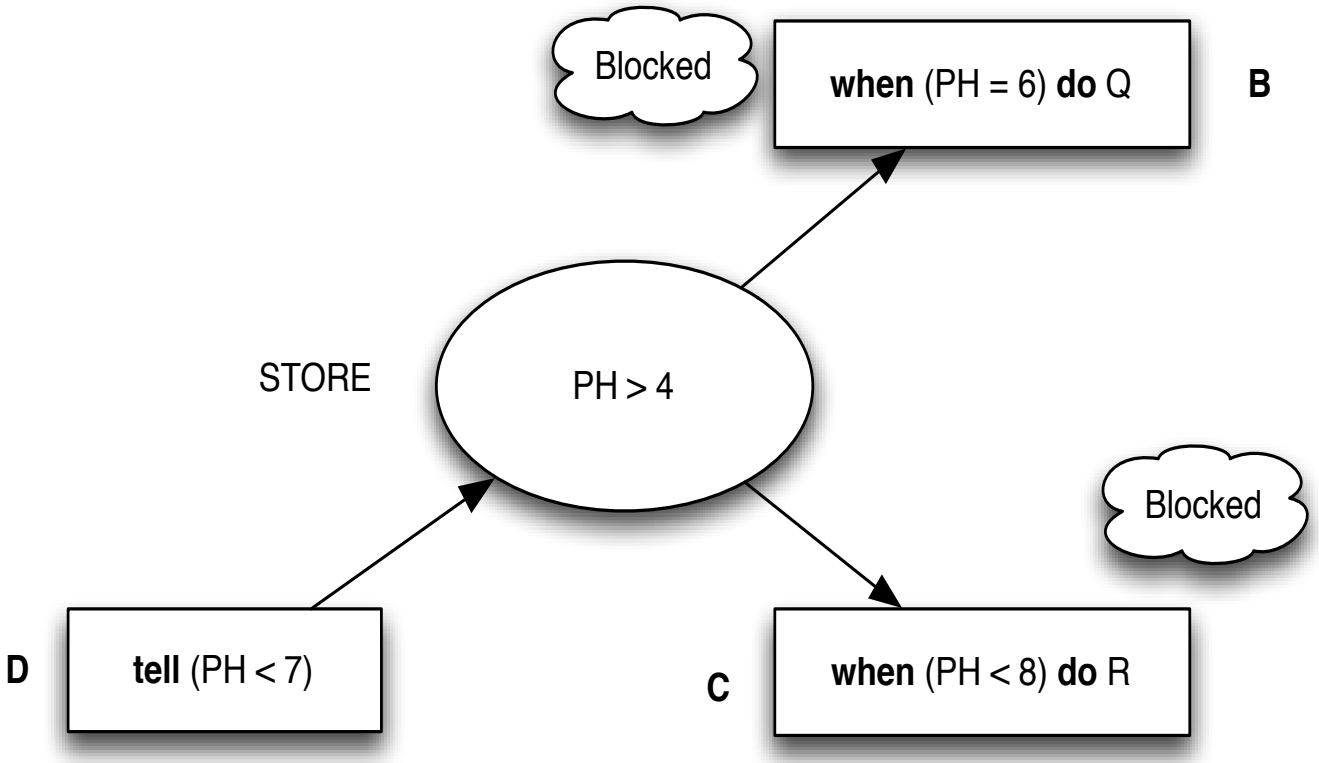
- tell agents add information to the store
- ask agents deduce information from the store

A computation is the activity resulting from the interaction of these two types of agents

Computing with partial information in CCP



Computing with partial information in CCP



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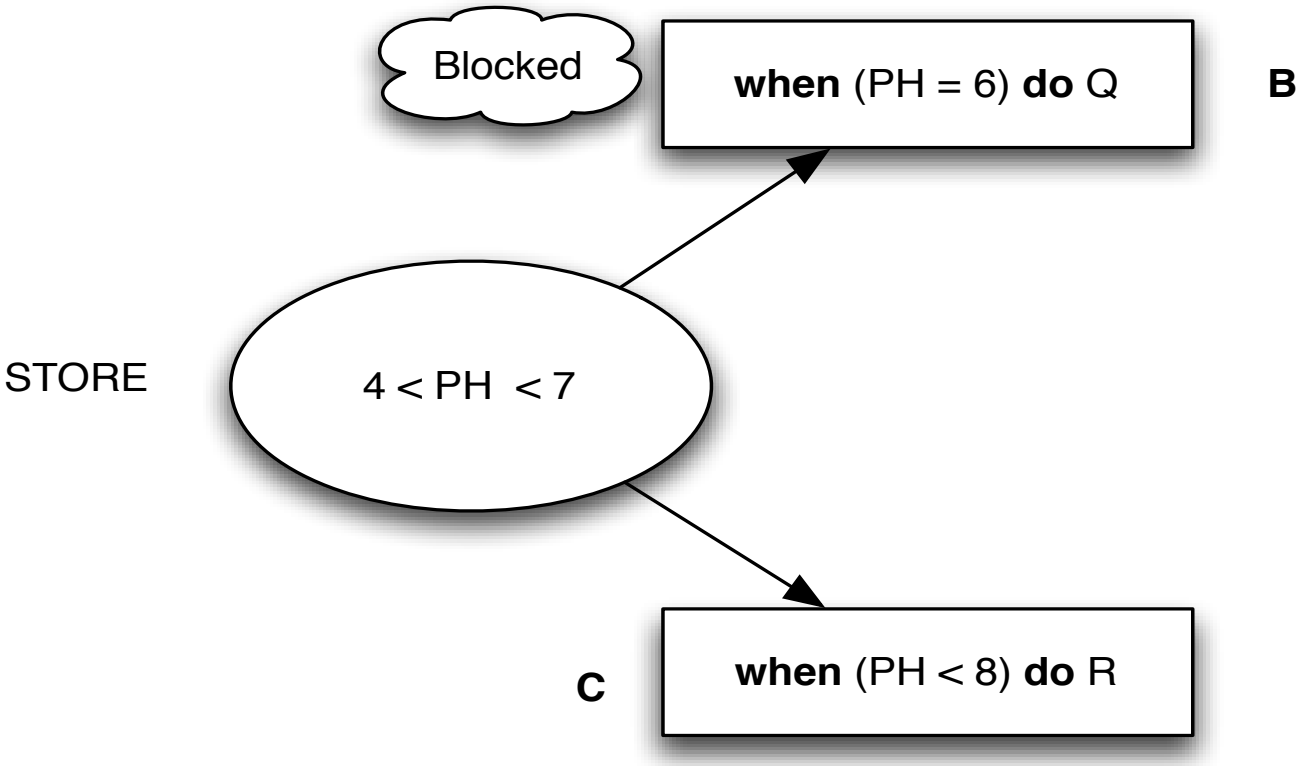
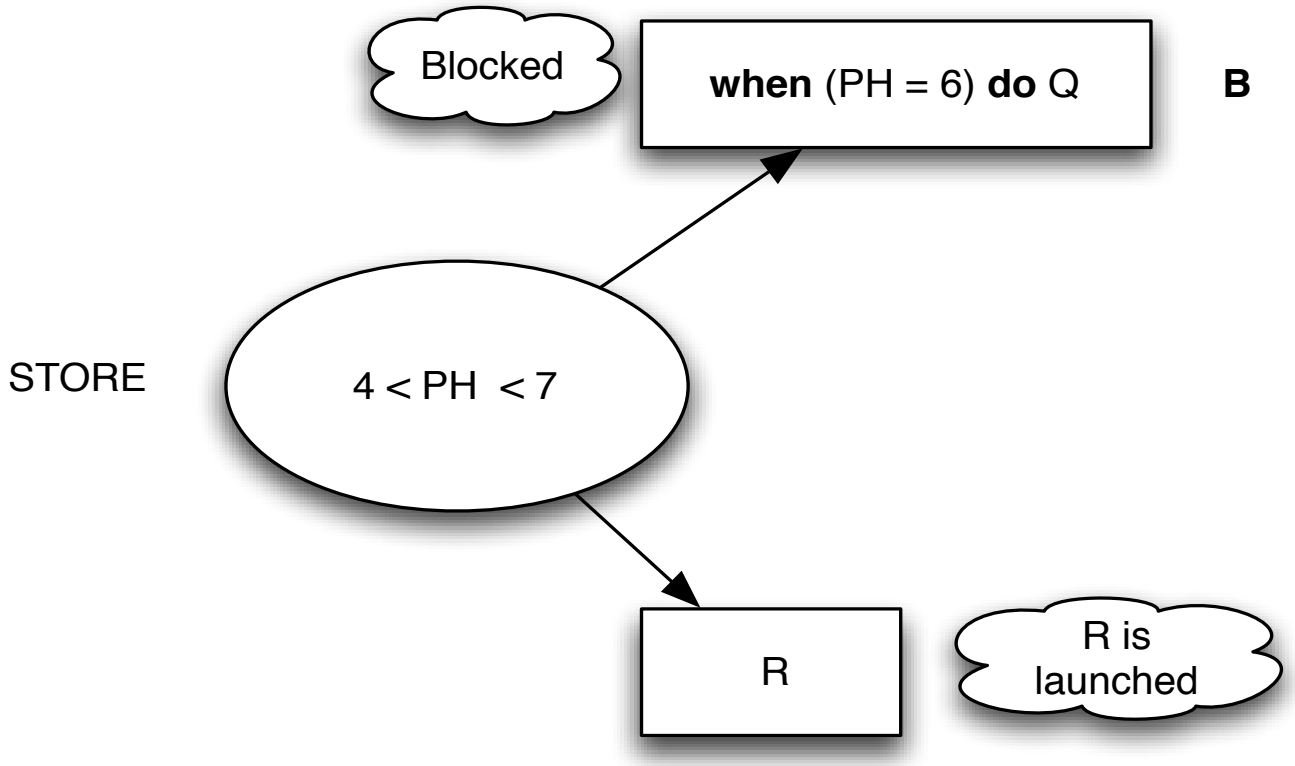


Computing with partial information in CCP



Concurrency

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- that they compute simultaneously?
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- that they compute simultaneously?
  A single observer (one absolute clock)?
  all observers read the same time for *A* and *B* in their clocks?
-

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What does it mean agents *A* and *B* are concurrent?

-
- that they compute in some (unknown) order?

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Concurrency

What does it mean agents *A* and *B* are concurrent?

-
- that they compute in some (unknown) order?
  the order is the same for all observers?
  the computation is the same no matter the order?

The standard ccp model

- accumulation of concurrent tells
  $tell\ (x > 3) \parallel tell\ (y \neq 0) \equiv tell\ (y \neq 0) \parallel tell\ (x > 3)$
  $\ldots \equiv tell\ (x > 3 \wedge y \neq 0)$
- interleaving of asks and tells
  $ask\ z < 1 \rightarrow P \parallel ask\ w = 5 \rightarrow Q$
  $\equiv$
  $ask\ z < 1 \rightarrow (P \parallel ask\ w = 5 \rightarrow Q\ )$
  $+$
  $ask\ w = 5 \rightarrow (\ Q \parallel ask\ z < 1 \rightarrow P\ )$
- grouping of asks
  $ask\ z < 1 \rightarrow\ (ask\ w = 5 \rightarrow P)$
  $\equiv\ \ ask\ (z < 1 \wedge w = 5) \rightarrow P$

Problems with the properties

nice properties, but:

they combine in such a way that intuitions do not scale up!

doing *A* and *B* concurrently might be intuitively the same as doing them sequentially in any order when *A* and *B* are not complex agents.

intuition accepts interleaving only for atomic processes

The determinate language

- A program is a declaration followed by an agent
  $P ::= L.A$
- Procedure declarations
  $L ::= \epsilon \mid p(X) :: A \mid L.L$
- Agents, $A ::=$
  - tell
    c
  - ask
    $c \rightarrow A$
  - Concurrent composition
    $A \parallel B$
  - Information hiding
    $\exists X.A$
  - procedure call
    $p(X)$

The determinate language: example

$$
\begin{array}{l}
append(In1, In2, Out) :: \\
\quad In1 = \lambda \rightarrow Out - In2 \\
\| \\
\quad c(In1) \rightarrow \exists X \; (Out = a(f(In1), X) \parallel append(r(In1), In2, X)
\end{array}
$$

Semantics: basic notions

- A set D of tokens or basic constraints
- $\vdash$, a decidable compact entailment relation
- non basic constraints are sets of tokens closed under entailment
  - $|D|$ is the set of finite non basic constraints
  - $c \in |D|$ implies that when $c \vdash P$ then $P \in c$
- $u \vdash P$ means: from constraint u, (basic) constraint P can be inferred

Semantics of the determinate language

- the only thing that deserves being observed is the store
- processes add information to the current store
- so they transform one store into another
- Since the language is determinate, transformation is a function:
  $f : |D| \rightarrow |D|$

Processes as constraint transformers

Say process *f* transforms constraint *c* (the store) into *d* and then stops. i.e. constraint *d* is a resting point.

- store *d* has at least as much information as *c*. i.e. $d \geq c$ (extensive)
- then *f* transforms *d* into *d* (idempotent)
- information is never substracted. i.e. $c \leq d \Rightarrow f(c) \leq f(d)$ (monotonicity)

Each language construct is a constraint transformer

Constraint transformers of tells and asks

We will focus only on ask and tell processes

- resting points of a $tell(c)$ is the set of constraints stronger than c

$$tell(c) = \{d \in |D| \mid d \geq c\}$$

- resting points of pprocess $c \rightarrow f$ is the set of constraints in the resting points of f that are stronger than c

$$(c \rightarrow f) = \{d \in |D| \mid d \geq c \Rightarrow d \in f\}$$

notice that constraints weaker than c are also included

Constraint transformers of parallel composition

- resting points of $P \parallel Q$ is the intersection of the resting points of P and Q

$$P \parallel Q = f_P \cap f_Q$$

Nondeterminate language: choice

$$\textit{choice}: \boxed{c_1 \rightarrow A_1 \ + \ c_2 \rightarrow A_2 \ + \ \ldots + \ c_n \rightarrow A_n}$$

In a store d, process

$$\sum_{j \in J} (c_j \rightarrow A_j)$$

transforms into

$$A_j \qquad \textit{if } d \geq c_j, \textit{ for some } j \in J$$

The store does not change.

choice

- choice adds expressivity to the language
- is at the heart of models in many applications
- but. . . it complicates semantics

For a given store, in a choice construct,

- many guards could be enabled
- computation may proceed along different paths

therefore,

processes are no longer constraints transformation functions

Semantics: idea

Since computation paths are important, for each process

- as before, record its resting points
- but also the interactions to get there

The idea is to keep a trace

$$a_1 \rightarrow (b_1 \parallel (a_2 \rightarrow b_2 \ldots (a_n \rightarrow b_n) \ldots)$$

of the path to reach resting point $c = b_1 \wedge \ldots \wedge b_n$. A process is thus

A set of pairs (t, c),
one for each resting point c

Semantics: trace operators

traces can be represented as bounded closure operators (f, u):

- u is a resting point of f (i.e. $u \in f$)
- the bound (i.e. domain) of f is $\downarrow u$

bco's can be ordered

$$(f, u) \leq (g, u) \text{ if } f \supseteq g$$

the (standard) semantics of a ccp process will then be

a set of bounded closure operators

True concurrency

for true concurrency

- the accummulation laws should not hold
- the interleaving law should not hold

observations should be more fine-grained:

- observe just tokens (i.e. basic constraints) in the store
- and also the conditions enabling them to get there

The observations

- Observations are contexted stores
- A contexted store is a set of contexted tokens a^u

  in the presence of tokens in u, token a is output

- contexted tokens are represented by bounded closure operators

Example

not(Modo, In, Out)::
$(Modo = ok \rightarrow Out = not(In)$
$+ Modo = trabado\_en\_1 \rightarrow Out = 1$
$+ Modo = trabado\_en\_0 \rightarrow Out = 0)$

Some contexted tokens in the presence of $M1 = ok, M2 = ok, X1 = 0, X2 = 1$ are:

$$Y1 = not(X1)^{M1=ok}$$
$$Y1 = 1^{M1=ok,X1=0}$$

And their bounded closure operators:

$$(M1 = ok \rightarrow not(X1), \downarrow not(X1))$$
$$(M1 = ok \rightarrow (X1 = 0 \rightarrow Y1 = 1), \downarrow Y1 = 1)$$

Semantics (Saraswat)

- Tell:

$$[\![tell(a)]\!] : \boxed{\{(f,u)|\ a \in u \wedge f \supseteq \tilde{a}\uparrow\}}$$

- parallel composition

$$[\![P \parallel Q]\!] : \boxed{\{(f \cap g, u)|\ (f,u) \in [\![P]\!] \wedge (g,u) \in [\![Q]\!]\}}$$

- choice,
  $[\![\sum_{j\in J}(c_j \rightarrow A_j)]\!]$:

$$\boxed{\begin{array}{l}\{(u\downarrow, u)|\ \forall j \in J.a_j \notin u\} \\ \cup \\ \bigcup_{j\in J}\{(f,u)|\ a_j \in u \wedge \exists (f_j,u) \in [\![A_j]\!] \wedge f \supseteq a_j \rightarrow f_j\}\end{array}}$$

Considerations about this semantics

- uses eventual tell
- extends nicely from the standard ccp semantics
- laws of interleaving and grouping do not hold

but...

- can it be extended to temporal ccp?
- can it be extended with probabilistic constructs?
- how about atomic tell?

Event structures

In a very general sense what is going on in a concurrent system?

- events occur in some determined or unknown order
- each event performs an action
- the actions performed change the state the system is in

event structures is a framework to represent the above in a precise way

Event structures: definition

An (prime) event structure is composed of

- a set A of events
- a causality (or enabling) relation "$\leq$" between events
- a conflict relation "$\#$" between events

with the properties:

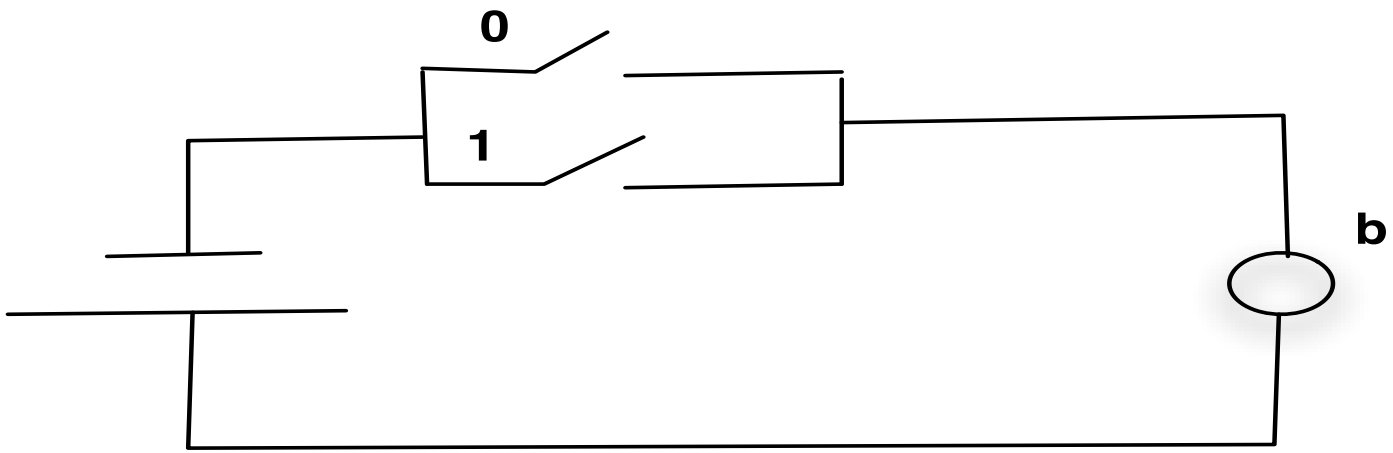
- $(A, \leq)$ is a partial order
- $\#$ is irreflexive and symmetric
- for all events $a, b, c \in A$,
  if a is in conflict with b and b causes c, then a is in conflict with c. i.e.,
  $a \# b \wedge b \leq c \Rightarrow a \# c$

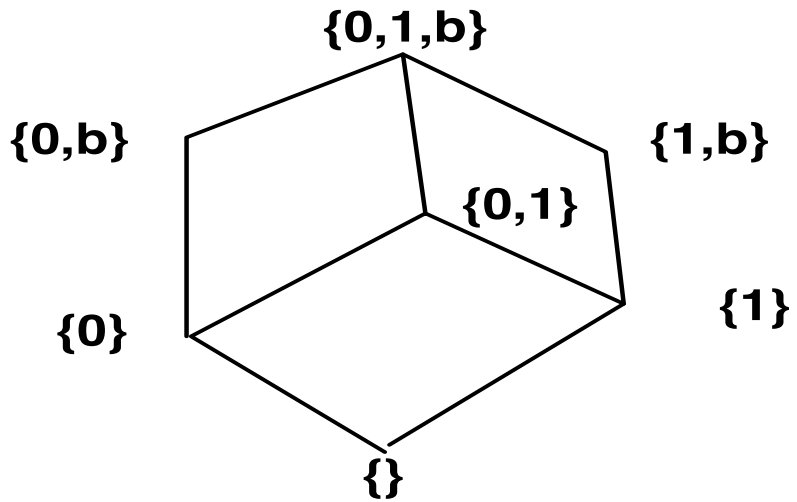
Event structures: the states of a system

States of a system are called configurations of an event structure

- configurations of a system are coherent sets of events
- events in a state mean that those events have already occurred when reaching that state
- therefore, if event b is in state s, and event a enables b, event a must also be in s
- states cannot have conflicting events:
  if $a \in s$ and $a \# b$ then $b \notin s$

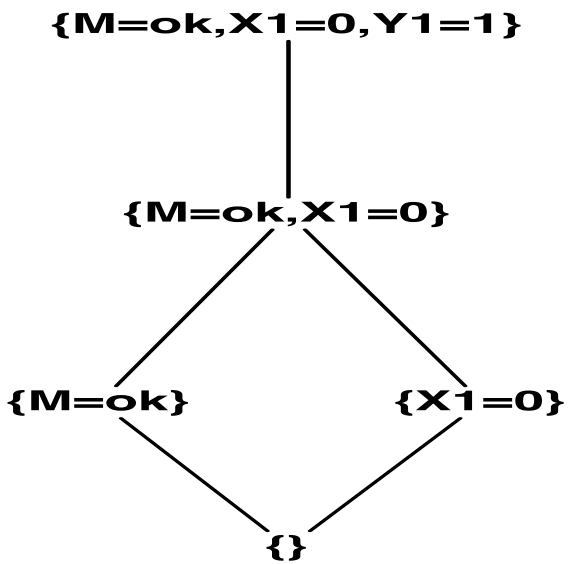
Event structures: an example





Event structures: an example (2)

Contexted tokens can be represented as event structures.
for example,
$(Y1 = 1)^{M1=ok,X1=0}$



so in ccp, events could be tokens, ordered by context?

Event structures and contexted stores

In a more precise way,

- events are occurrence of actions
- the action of each event is a set of contexted stores
- contexted stores are sets of tokens

For example,

$$\mathit{ask}\ M = \mathit{ok} \rightarrow \mathit{ask}\ X = 0 \rightarrow Y = 1$$

has events $E = \{e_1, e_2\}$ and

$$\begin{array}{l} \mathit{action}(e_2) = \ (Y = 1)^{\{\{X=0\}\}} \\ \mathit{action}(e_1) = \ (Y = 1)^{\{\{M=\mathit{ok},X=0\}\}} \end{array}$$

Representation of event structures: chu spaces

A chu space or "couple" is a matrix (A, X, Σ)

- Set A, the rows, is a set of events
- Set X, the columns, is a set of states

when $\Sigma = \{0, 1\}$,

- A system as a chu space gives, for each state, the events that have occurred
- dually, for each event, the states in which it has occured

Both events and states can then be regarded as functions

$a(s) = 1$ means "event a has occurred in state s"

or, equivalently,

$s(a) = 1$ means "event a belongs to configuration s"

Representation of event structures: chu spaces (2)

An event structure can be easily recovered from a chu space:

- given a chu space $\mathbb{S} = (A, X)$ with $x \in A \to \{0, 1\}$
- the event structure representd by $\mathbb{S}$ is $(A, \leq, \#)$ where
- $a \leq b$ if, in every state s, when b is in s, a is also in s
- and $a \# b$ if, in all states s,
  - when a is in s, event b is not in s,
  - and, when b is in s, then a is not in s

chu space of **tell** and **ask**

$tell(a)$: e_a | 0 1 | $act(e_a) = a^{\{\emptyset\}}$

$ask\ a \rightarrow b$: e_b | 0 1 | $act(e_b) = b^{\{\{a\}\}}$

chu space of composition and choice

$tett(a) \parallel tell(b)$:

e_a	0	1	0	1
e_b	0	0	1	1

$\{\{\}, \{a\}, \{b\}, \{a, b\}\}$

$tell(a) + tell(b)$:

e_a	0	1	0
e_b	0	0	1

$\{\{\}, \{a\}, \{b\}\}, \quad a \# b$

Defining combinators

- tell(a) simply adds contexted store $a^{\{\emptyset\}}$
- $a \rightarrow b$ adds contexted store $b^{\{\{a\}\}}$
- parallel composition $P \parallel Q$ states all possible combinations of contexted stores in P and Q
- choice $P + Q$ filters $P \parallel Q$ by precluding non common tokens of P and Q to be in the same configuration

Defining composition

$[\![P \parallel Q]\!] \triangleq (A \cup B, Z)$, where

- $Z = \{z \in 2^{A \cup B} \mid z \downarrow_A \in X \wedge z \downarrow_B \in Y\}$
- $z \downarrow_A$ is the restriction of the domain of z to the set A

so,

- for tokens (events) $a \in A, a \notin B$ and $b \in B, b \notin A$, states of z contain $\{a\}, \{b\}$ and $\{a, b\}$
- and for common events of A and B, compatible states are kept.

Since each event is unique, the last option never arises.
Actions of each event carry on to the composition

Defining composition: example

$$(tell(a) \parallel tell(b)) \parallel (tell(b) + tell(c))$$

$a \parallel b$:

e_a :	0	1	0	1
e_b :	0	0	1	1

$b + c$:

e'_b	0	1	0
e_c	0	0	1

$action(e'_b) = b^{\{\emptyset\}}$

$(a \parallel b) \parallel (b + c)$ ::

e_a :	0	1	0	1	0	0	1	1	0	0	1	1
e_b :	0	0	1	1	0	0	0	0	1	1	1	1
e'_b :	0	0	0	0	1	0	1	0	1	0	1	0
e_c :	0	0	0	0	0	1	0	1	0	1	0	1

Notice that four of the above 12 states are redundant

Defining choice

Let the sets of events of P be A and those of Q be B
the event structure of $P + Q$ contains

- states where neither A nor B have occurred

$$\overline{A \cup B}$$

- states of P, together with non occurrence of those events in B not in A

$$P \parallel \overline{B - A}$$

- states of Q, together with non occurrence of those events in A not in B

$$Q \parallel \overline{A - B}$$

Defining choice: formalization

Given a set of events $A = \{a_1, \ldots, a_n\}$, let $\overline{A}$ be the event structure

$$\begin{array}{c|c} a_1 & 0 \\ \ldots & \ldots \\ a_n & 0 \end{array} \qquad \textit{i.e.}(A, \leq, \#) \textit{ with configurations } \{\,\{\}\,\}$$

the semantics of choice is

$$[\![P + Q]\!] \triangleq \overline{A \cup B} \vee (P \parallel \overline{B - A}) \vee (Q \parallel \overline{A - B})$$

where disjunction of processes $P \vee Q$ is defined as

$$(A \cup B, \{z \in 2^{A \cup B} \mid z \downarrow_A \in X \vee z \downarrow_B \in Y\}$$

Defining ask

the structure for $a \rightarrow Q$ contains states of Q but

- each contexted action of Q is augmented with $\{a\}$

formally:

$$[\![a \rightarrow Q]\!] \triangleq [\![Q]\!]$$

and , for each event e of $[\![Q]\!]$ such that $action(e) = b^w$, set

$$action'(e) = b^{w'}$$

where $w' = \{z \cup \{a\} \mid z \in w\}$

Where is the store?

At each state the store is the conjunction of tokens corresponding to event entries equal to 1
but,

what is the definition of conjunction for contexted stores?

A little algebra of contexted store reductions,

$$a^x \wedge a^y \longrightarrow a^{x \cup y}$$
$$a^x \longrightarrow a^{x \setminus \{s\}} \quad \text{if there exists } u \in x \text{ such that } u \subseteq s$$
$$a^x \wedge b^y \longrightarrow b^y, a^{x \setminus \{u\}} \quad \text{if there exists } s \in y \text{ such that } s \cup \{b\} \subseteq u$$

in particular, $a^{\{\emptyset\}} \wedge a^x \longrightarrow a^{\{\emptyset\}}$ and
$a^{\{b\}} \wedge b^{\{\emptyset\}} \longrightarrow b^{\{\emptyset\}}, a^{\{\emptyset\}}$

The store

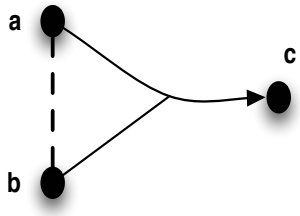
Let s be a state.
Let $\mathcal{C}$ be the colection of events a such that$s(a) = 1$
Let $Clos(\mathcal{C})$ be the contexted stores resulting from applying the reductions to actions of $\mathcal{C}$

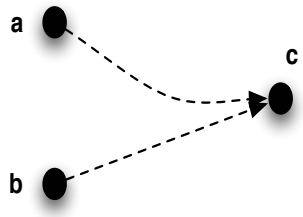
$$store(s) = \boxed{\{\hat{b} \mid b^{\{\emptyset\}} \in Clos(\mathcal{C})\}}$$

where $\hat{b}$ is the entailment closure of b

timed event structures

prime event structures are extended to bundle event structures:

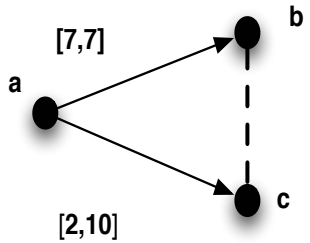


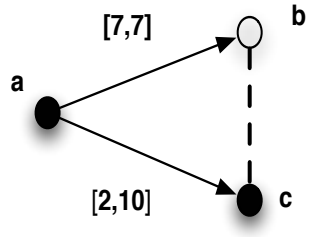


- A set of events
- an irreflexive conflict relation
- a bundle relation

timed event structures

bundle event structures are extended with temporal labels:





- A set of events
- an irreflexive conflict relation
- a bundle relation
- a labelling function
- an event delay function
- a bundle delay function
- a set of urgent events

atomic tell?

we have seen eventual tell. The atomic tell,

- implements test and set operation
- the store can never be inconsistent

$tell(x = 3) \parallel tell(x > 7)$ may reduce to

$$< skip, x = 3 >$$

it would be nice to...

- work out in detail the eventual tell chu-space semantics of ccp
- then the atomic tell
- then integrate temporal labels into the chu-space framework